Calculus & Linear Algebra — Reference Sheet

Derivatives

$\frac{d}{dx}x^n = nx^{n-1}$ — power rule. $\frac{d}{dx}e^x = e^x$ — exponential is its own derivative. $\frac{d}{dx}\ln x = \frac{1}{x}$ for $x > 0$.

Product & Chain Rules

$$(fg)' = f'g + fg', \qquad (f \circ g)'(x) = f'(g(x))\, g'(x).$$

Integrals

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1), \qquad \int \frac{1}{x}\, dx = \ln|x| + C.$$

Integration by parts: $\int u\, dv = uv - \int v\, du$.

Taylor Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

Matrix Basics

$(AB)^\top = B^\top A^\top$ — transpose reverses order. $(AB)^{-1} = B^{-1}A^{-1}$ when both invertible.

Eigenvalues

$Av = \lambda v$ with $v \neq 0$; solve $\det(A - \lambda I) = 0$ for λ.

Determinant & Trace

$\det(A) = \prod_i \lambda_i$, $\quad \operatorname{tr}(A) = \sum_i \lambda_i = \sum_i A_{ii}$.

Norms

$\|x\|_2 = \sqrt{\sum_i x_i^2}$, $\quad \|x\|_1 = \sum_i |x_i|$, $\quad$ Cauchy–Schwarz: $|x \cdot y| \leq \|x\|_2 \|y\|_2$.

Gradient & Hessian

$\nabla f(x) \in \mathbb{R}^n$ points in the direction of steepest ascent; $H_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}$ describes local curvature.